Design example: Wood diaphragm using envelope method

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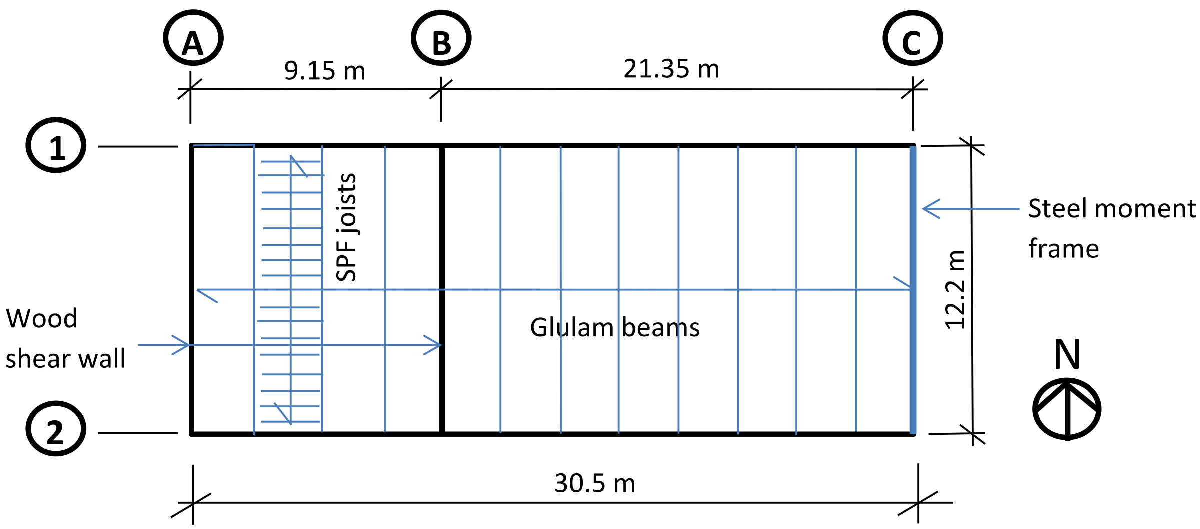
INTRODUCTION

This building is a typical one-storey commercial building located in Vancouver, BC. The plan dimensions are 30.5 m x 12.2 m (100' x 40'), with a building height of 5 m. The walls are wood-based shearwalls, with a wood diaphragm roof and a steel moment frame at the storefront. The roof plan is shown in Figure 1.

The site is Seismic Class 'C'. Wind, snow and seismic figures specific to the project location are taken from the current version of the British Columbia Building Code (2012).

Roof dead load is assumed to be 1.0 kPa and the wall weight is 0.5 kPa. The weight of non-structural items including mechanical equipment and the storefront façade has not been included in this example for simplicity.



Derivation of Force

Snow Load:

$S_s = 1.8\ kPa$, $S_r = 0.2\ kPa$ [NBCC, Appendix C, Table C-2]

$I_s = 0.9\ (SLS)$, $I_s = 1.0\ (ULS)$ [NBCC, Table 4.1.6.2]

$$S = I_s[S_s(C_bC_wC_sC_a) + S_r]$$

$$S_{SLS} = 0.9 \times (1.8 \times 0.8 \times 1.0 \times 1.0 \times 1.0 + 0.2) = 1.48\ kPa$$

$$S_{ULS} = 1.0 \times (1.8 \times 0.8 \times 1.0 \times 1.0 \times 1.0 + 0.2) = 1.64\ kPa$$

Lateral Forces in N-S Direction

Wind Load:

$I_w = 1.0 \ (ULS)$ [NBCC, Appendix C, Table C-2]

$q = 0.48 \ kPa$ [NBCC, Appendix C, Table C-2]

$C_p = 0.75$ [NBCC Commentary]

$C_g = 2.0$ [NBCC Commentary]

$C_e = (5.0/10)^{0.2} = 0.87$ (0.9 minimum is used) [NBCC 4.1.7.1 .5a]

$p = I_w q C_e C_g C_p = 1.0 \times 0.48 \times 0.9 \times 0.75 \times 2.0 = 0.65 \ kPa$ [NBCC 4.1.7.1]

$N_{w,N-S} = 1.4 \times (0.65 \times 5 \times 30.5) = 140 \ kN$

Seismic Load:

$S_a(0.2) = 0.94$ [NBCC, Appendix C, Table C-2]

$I_E = 1.0 \ (ULS)$ for Normal building, Site Class 'C'

Roof

$$W_R = 0.25 \ Snow \ load + 1.0 \ Dead \ load$$
$$= 0.25 \times 1.64 \times 12.2 \times 30.5 + 1.0 \times 1.0 \times 12.2 \times 30.5 = 524 \ kN$$

Walls

$$W_{W,N-S} = weight \ of \ half \ height \ of \ perpendicular \ walls + weight \ of \ full \ height \ of \ parallel \ walls$$
$$= 1/2 \times 5.0 \times 30.5 \times 0.5 \times 2 + 5 \times 12.2 \times 0.5 \times 2 = 76 + 61 = 137 \ kN$$

Total Seismic weight

$W_{N-S} = W_R + W_{W,N-S} = 524 + 137 = 661 \ kN$

$T_a = 0.05 \times h_n^{3/4}$ for shear wall and other structures [NBCC 4.1.8.11.3c]

$= 0.05 \times (5.0)^{3/4} = 0.17 \ s$

$S(T) = F_a S_a(0.2)$ for T ≤ 0.2s [NBCC 4.1.8.4.7]

$= 1.0 \times 0.94 = 0.94$

According to Clause 4.1.8.9.3, for combinations of different types of SFRS acting in the same direction in the same storey, $R_d R_o$ shall be taken as the lowest value of $R_d R_o$ corresponding to these systems.

$$R_d R_o = \min(R_d R_{o(wood \ shear \ wall)}, R_d R_{o(steel \ moment \ frame)})$$
$$= \min(3.0 \times 1.7, \ 1.5 \times 1.3) = 1.5 \times 1.3$$

$$V_{E,N-S} = \frac{2/3 \times S(0.2) \times I_E \times W}{R_d R_o}$$ [NBCC 4.1.8.11.2c]

$$= \frac{2/3 \times 0.94 \times 1.0 \times 661}{1.5 \times 1.3} = 212 \ kN$$

Therefore, the governing lateral force in North-South direction is the seismic load.

Lateral Forces in E-W Direction

Wind Load:

$$N_{w,E-W} = 1.4 \times (0.65 \times 5 \times 12.2) = 55 \ kN$$

Seismic Load:

Walls

$$W_{W,E-W} = weight\ of\ half\ height\ of\ perpendicular\ walls + weight\ of\ full\ height\ of\ parallel\ walls$$

$$= 1/2 \times 5.0 \times 12.2 \times 0.5 \times 2 + 5 \times 30.5 \times 0.5 \times 2 = 31 + 152 = 183 \ kN$$

Total seismic weight

$$W_{E-W} = W_R + W_{W,E-W} = 524 + 183 = 707 \ kN$$

$R_d R_o = 3.0 \times 1.7$ for wood shear walls

$$V_{E,E-W} = \frac{2/3 \times S(0.2) \times I_E \times W}{R_d R_o}$$ [NBCC 4.1.8.11.2c]

$$= \frac{2/3 \times 0.94 \times 1.0 \times 707}{3.0 \times 1.7} = 87 \ kN$$

Therefore, the governing lateral force in East-West direction is the seismic load.

Shear wall design with envelope method

Diaphragms are typically designed assuming that the diaphragm is flexible, spanning between shear walls like a simply supported beam. However, the assumption of flexible diaphragm is not always valid and could lead to unconservative design if used in the wrong circumstances. In reality, wood diaphragms fall somewhere between the flexible and rigid behaviour. In this case, envelope approach which takes the highest forces from rigid and flexible assumptions can be used as a conservative estimation. This approach is recommended in situations where it is difficult to estimate relative stiffness of the lateral force-resisting system and diaphragms.

Design of shear walls with flexible diaphragm assumption

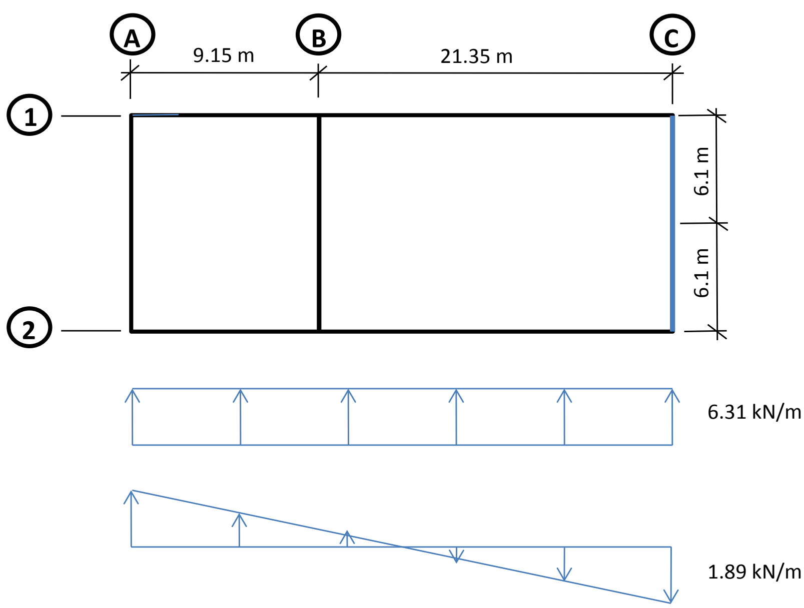
N-S direction

It is assumed that the lateral force from the weight of the top half of the walls perpendicular to the lateral load direction is resisted by the diaphragm and is redistributed to shearwalls based on diaphragm flexibility. The lateral force from the full weight of the wall parallel to the lateral load direction is resisted by the wall itself.

Assuming seismic force from the roof and perpendicular walls is equally distributed along the building length

$$v_{N-S} = \frac{212 \times (661 - 61) / 661}{30.5} = 6.31 \ kN/m$$

For flexible diaphragms, an accidental eccentricity of 5% of diaphragm dimension perpendicular to applied force should be taken into account and the largest seismic force should be used in the design of each vertical element (Commentary J in NBCC-2010 Structural Commentaries).



Shear wall at Gridline A

$$V_A = 1.89 \times \left(1 + \frac{30.5/2 - 9.15/2}{30.5/2}\right) \times \frac{9.15}{2} \times \frac{1}{2} + 6.31 \times \frac{9.15}{2} + 212 \times \frac{61/2}{661} = 46.0 \ kN$$

$$v_A = 46.0/12.2 = 3.77 \ kN/m$$

The shear wall consisting of SPF framing member and 12.5 mm plywood with 8d nails (d = 3.25 mm) spaced at 150 mm o.c. around panel edges is selected. The specified shear strength, v_d, is 7.1 kN/m and the factored lateral load resistance is:

$$v_{r,A} = \phi v_d K_D K_{SF} J_{ub} J_{sp} J_{hd} = 0.7 \times 7.1 \times 1.15 \times 1.0 \times 1.0 \times 0.8 \times 1.0 = 4.57 \ kN/m > 3.77 \ kN/m$$

Shear wall at Gridline B

$$V_B = 1.89 \times \frac{(30.5/2 - 9.15/2)^2}{30.5/2} \times \frac{1}{2} - 1.89 \times \frac{(30.5/2 - 21.5/2)^2}{30.5/2} \times \frac{1}{2} + 6.31 \times \left(\frac{9.15}{2} + \frac{21.35}{2}\right) + 212 \times \frac{61/2}{661}$$
$$= 111.8 \ kN$$

$$v_B = 111.8/12.2 = 9.16 \ kN/m$$

The shear wall consisting of SPF framing member and 12.5 mm plywood with 8d nails (d = 3.25 mm) spaced at 50 mm o.c. around panel edges is selected. The specified shear strength, v_d, is 17.4 kN/m and the factored lateral load resistance is:

$$v_{r,B} = \phi v_d K_D K_{SF} J_{ub} J_{sp} J_{hd} = 0.7 \times 17.4 \times 1.15 \times 1.0 \times 1.0 \times 0.8 \times 1.0 = 11.2 \ kN/m > 9.16 \ kN/m$$

Shear wall at Gridline C

$$V_C = 1.89 \times \left(1 + \frac{30.5/2 - 21.35/2}{30.5/2}\right) \times \frac{21.35}{2} \times \frac{1}{2} + 6.31 \times \frac{21.35}{2} = 80.5 \ kN$$

$$v_C = 80.5/12.2 = 6.60 \ kN/m$$

A steel moment frame with W360x196 columns and W530x219 beam was selected based on preliminary sizing of the steel members and investigation of the displacement versus force in a proprietary 2D analysis software program.

E-W direction

As there are only two walls in the East-West direction, each wall take 50% of the lateral load plus an additional 5% eccentric load.

Shear wall at Gridline 1 and 2

$$V_1 = V_2 = \frac{87\times(707-152)/707}{2}\times(0.55/0.5)+87\times((152/2)/707) = 46.9\ kN$$

$$v_1 = v_2 = 46.9/30.5 = 1.54\ kN/m$$

The shear wall consisting of SPF framing member and 12.5 mm plywood with 8d nails (d = 3.25 mm) spaced at 150 mm o.c. around panel edges is selected. The specified shear strength, v_d, is 7.1 kN/m and the factored lateral load resistance is:

$$v_{r,1} = v_{r,2} = \phi v_d K_D K_{SF} J_{ub} J_{sp} J_{hd} = 0.7\times7.1\times1.15\times1.0\times1.0\times0.8\times1.0 = 4.57\ kN/m > 1.54\ kN/m$$

Design of shear walls with rigid diaphragm assumption

For a diaphragm that is designated as rigid, the lateral force is distributed to the supporting shear walls according to their relative stiffness, with consideration of additional shear force due to torsion for seismic design. In addition to the torsion due to natural eccentricity, NBCC requires that a minimum eccentricity of 10% of the plan dimension of the building perpendicular to the direction of seismic load should also be considered.

In order to determine the shear walls stiffness, the shear walls obtained from the flexible diaphragm design are used as the initial input.

The shear wall stiffness can be determined as follows:

$$k = \frac{F}{\Delta} = \frac{vL}{\dfrac{2vH^2}{3EAL}+\dfrac{vH}{B_v}+0.0025He_n+\dfrac{H}{L}d_a}$$

where

v = maximum shear due to specified loads at the top of the wall, N/mm

H = height of shearwall segment, mm

E = elastic modulus of boundary element, N/mm^2

A = cross-sectional area of the boundary member, mm^2

L = length of shearwall segment, mm

B_v = shear through-thickness-rigidity of the sheathing, N/mm [Table 7.3A - 7.3C, CSA O86]

e_n = nail deformation, mm [Clause A.9.7, CSA O86]

d_a = total vertical elongation of the wall anchorage system (including fastener slip, device elongation, anchor or rod elongation, etc.) at the induced shear load

The stiffness's of the shear walls are listed in the table below:

Shear Wall	v [N/mm]	A [mm^2]	L [mm]	B_v [N/mm]	Force/nail [N]	e_n [mm]	d_a [mm]	Δ [mm]	k [kN/m]
A	3.77	10640	12200	5700	566	0.40	1.89	9.06	5074
B	9.16	10640	12200	5700	458	0.28	4.59	13.44	8312
C	6.60	N/A	N/A	N/A	N/A	N/A	N/A	18.99	4240
1	1.54	10640	30500	5700	231	0.12	0.77	2.92	16079
2	1.54	10640	30500	5700	231	0.12	0.77	2.92	16079

Note:

1) End studs of shearwalls in Gridline A and B are made of two 2x6's (76 x 140 mm).
2) E = 9500 N/mm^2
3) e_n: Force per nail = v x nail spacing $\rightarrow$ e_n for 8d common nail (d = 3.25 mm) from Table A. 9.7 of CSA-O86
4) d_a is calculated based on the percentage utilization of capacity of HHDQ11 hold-down (by Simpson) at each end of shear wall. For HHDQ11 hold-down, deflection = 5.54 mm at capacity of 55.25 kN.
   a) For shear wall at gridline A, d_a = (3.77 x 5) / 55.25 x 5.54 = 1.89 mm
   b) For shear wall at gridline B, d_a = (9.16 x 5) / 55.25 x 5.54 = 4.59 mm
   c) For shear wall at gridline 1 and 2, d_a = (1.54 x 5) /55.25 x 5.54 = 0.77 mm
5) The force-displacement relationship, or stiffness, for the steel moment frame was obtained from a 2D analysis using a proprietary software program.

For rigid diaphragm, the lateral force distributed to supporting shear wall *i* can be determined as follows:

$$V_i = \frac{F \times k_i}{\sum k} + \frac{T \times k_i \times d_i}{J}$$

where

k = wall stiffness, N/mm

d = distance from the wall to the centre of rigidity (CoR), mm.

T = torsional moment

F = total lateral load on the supporting shear walls

$$J = \sum kd_x^2 + \sum kd_y^2$$

N-S direction

The center of mass (*CoM*) is located at half of the building length in the N-S and E-W direction. The centre of rigidity in the E-W direction ($CoR_{E\text{-}W}$) is coincident with the *CoM*. The centre of rigidity in the N-S direction ($CoR_{N\text{-}S}$) is determined as follows:

$$CoR_{N-S} = \frac{\sum k \cdot d}{\sum k} = \frac{5074 \times 0 + 8312 \times 9.15 + 4240 \times 30.5}{5074 + 8312 + 4240} = 11.65 \ m \text{ from Gridline A}$$

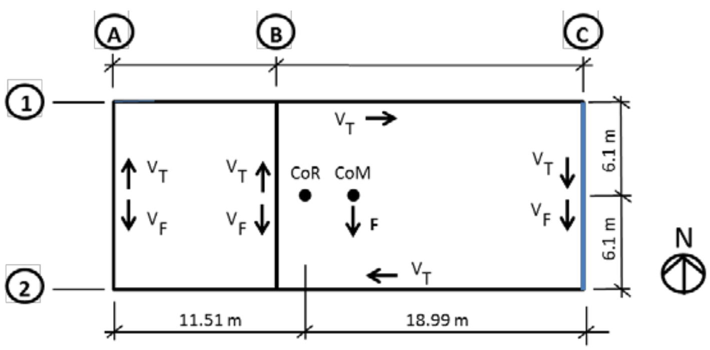
The torsion is determined as follows:

$$T_{N-S} = 212 \times (600/661) \times (30.5/2 - 11.65) \pm 212 \times (600/661) \times 0.1 \times 30.5 = 694 \pm 588 \ kN \cdot m$$

For shear walls in Gridlines A and B, the maximum lateral force on each shear wall is determined with $T_{N-S} = 694 - 588 = 106 \ kN \cdot m$.

For shear walls in Gridlines C, 1 and 2, the maximum lateral force on each wall is determined with $T_{N-S} = 694 + 588 = 1282 \ kN \cdot m$.

The force on supporting shear walls due to lateral load and eccentricity is shown below.



The lateral force on each wall is determined as follow:

Shear Wall	$K_{x,i}$ [kN/m]	$K_{y,i}$ [kN/m]	$d_{x,i}$ [m]	$d_{y,i}$ [m]	$kd_{x,i}^2$ [kN·m]	$Kd_{y,i}^2$ [kN·m]	V_i [kN]	v_i [kN/m]
A	5074	-	11.65	-	688886	-	63.5	5.20
B	8312	-	2.50	-	52018	-	100.1	8.20
C	4240	-	18.85	-	1506301	-	76.1	6.24
1	-	16079	-	6.1	-	598298	36.5	1.20
2	-	16079	-	6.1	-	598298	36.5	1.20

As the factored shear resistance of shear wall in Gridline A is smaller than the shear force, nail spacing is revised to 100 mm o.c. along the panel edges to accommodate the increase in force. The specified shear strength, v_d, is 10.3 kN/m and the factored lateral load resistance is:

$$v_r = \phi v_d K_D K_{SF} J_{ub} J_{sp} J_{hd} = 0.7 \times 10.3 \times 1.15 \times 1.0 \times 1.0 \times 0.8 \times 1.0 = 6.63 \ kN/m \ > 5.20 \ kN/m$$

With the new shear wall configuration in Gridline A, the above process is repeated until the force distribution to each shear wall is converged. The table below shows the force distribution to all the shear walls during each iteration.

Shear Wall	Iteration 1		Iteration 2		Iteration 3		Iteration 4		Iteration 5	
	K_i [kN/m]	V_i [kN]	K_i [kN/m]	V_i [kN]	K_i [kN/m]	V_i [kN]	K_i [kN/m]	V_i [kN]	K_i [kN/m]	V_i [kN]
A	5074	63.5	6387	69.4	6202	68.3	6231	68.5	6226	68.5
B	8312	100.1	8431	94.1	8491	95.2	8481	95.1	8482	95.1
C	4240	76.1	4240	76.2	4240	76.0	4240	76.0	4240	76.0
1	16079	36.5	16079	39.1	16079	38.8	16079	38.9	16079	38.9
2	16079	36.5	16079	39.1	16079	38.8	16079	38.9	16079	38.9

E-W direction

The center of mass (CoM) is located at half of the building length in the E-W direction. The centre of rigidity (CoR) in the E-W direction is coincident with the CoM. Therefore torsion is due to the accidental torsion only.

The seismic force to be distributed to the shear walls in the E-W direction is calculated as follows:

$$F_{E-W} = 87 \times \frac{(707-152)}{707} = 68 \ kN$$

The torsion is determined as follows:

$$T_{E-W} = \pm 87 \times (555/707) \times 0.1 \times 12.2 = \pm 83 \ kN \cdot m$$

Using the shear walls stiffness obtained from the rigid diaphragm design in N-S direction as the initial input, the table below shows the convergence of shear force, *v*, for each shear wall.

Shear Wall	Iteration 1		Iteration 2		Iteration 3	
	K_i [kN/m]	V_i [kN]	K_i [kN/m]	V_i [kN]	K_i [kN/m]	V_i [kN]
A	6226	1.6	7144	1.7	7144	1.7
B	8482	0.3	8744	0.2	8744	0.2
C	4240	1.9	4240	1.9	4240	1.9
1	16079	45.7	16079	45.6	16079	45.6
2	16079	45.7	16079	45.6	16079	45.6

The table below summarises the calculated lateral force on each shear wall based on flexible and rigid diaphragm assumptions, with the highest force taken for envelope approach. This table also shows the difference in design force depending on which method is chosen.

Shear Wall	Flexible diaphragm		Rigid diaphragm		Envelope approach	
	V_i [kN]	v_i [kN/m]	V_i [kN]	v_i [kN/m]	V_i [kN]	v_i [kN/m]
A	46.0	3.77	68.5	5.61	68.5	5.61
B	111.8	9.16	95.1	7.80	111.8	9.16
C	80.5	6.60	76.0	6.23	80.5	6.60
1	46.9	1.54	45.6	1.50	46.9	1.54
2	46.9	1.54	45.6	1.50	46.9	1.54

Diaphragm design

N-S direction

Assuming that diaphragms are designed to yield before the supporting SFRS, the diaphragm shall be designed for seismic loads determined using the R_dR_o factors for the vertical SFRS according to Clause 9.8.5.2.1 of CSA O86. Such seismic design loads, however, shall not be less than loads determined using R_dR_o = 2.0. As the seismic load in N-S direction is determined using R_dR_o = 1.95, it meets this requirement.

The maximum shear force on the diaphragm can be determined from the reaction force at the supporting shear walls based on force equilibrium. Based on the maximum force on each shear wall using envelope approach, the maximum shear forces on diaphragm at Gridline A, B and C are obtained as follows:

$$V_{D,A} = 68.5 - 212 \times (61/2)/661 = 58.7\ kN \rightarrow v_{D,A} = 58.7/12.2 = 4.81\ kN/m$$

$$V_{D,B}^{-} = 6.31 \times 9.15 + 1.89 \times \left(1 + \frac{30.5/2 - 9.15}{30.5/2}\right) \times \frac{9.15}{2} - (46.0 - 212 \times (61/2)/661) = 33.6\ kN \rightarrow$$

$$v_{D,B}^{-} = 33.6/12.2 = 2.75\ kN/m$$

$$V_{D,B}^{+} = (111.8 - 212 \times (61/2)/661) - V_{D,B}^{-} = 68.4\ kN \rightarrow v_{D,B}^{+} = 68.4/12.2 = 5.61\ kN/m$$

$$V_{D,C} = 80.5\ kN \rightarrow v_{D,C} = 80.5/12.2 = 6.60\ kN/m$$

Select 38 mm x 235 mm SPF joists and 12.5 mm plywood with 3.66 mm nails at 100 mm o.c. along diaphragm boundaries (panel layout case 3), the factored shear strength is

$$v_r = \phi v_d K_D K_{SF} J_{sp} = 0.7 \times 10.5 \times 1.15 \times 1.0 \times 0.8 = 6.76\ kN/m > 6.60\ kN/m$$

E-W direction

The seismic force on supporting shear wall at Gridline 1 and 2 is

$$V_1 = V_2 = 46.9 - 87 \times (152/2)/707 = 37.5\ kN \rightarrow v_1 = v_2 = 37.5/30.5 = 1.23\ kN/m$$

In the E-W direction wood diaphragm is supported on wood shearwalls, and therefore in accordance with Clause 9.8.4.2 of CSA O86, the seismic design force for the diaphragm is taken as:

$$V_D = C_D \times F$$

where

C_D = the lesser of 1.2 and *C*, (C is the over-capacity coefficient for the shearwall, $C = v_r / v_f$)

= $\min(1.2,\ 4.57/1.23) = 1.2$

F = factored seismic force calculated using $R_d R_o$ for wood shearwalls

Therefore

$$V_D = 1.2 \times 37.5 = 45.1\ kN \rightarrow v_D = 45.1/30.5 = 1.48\ kN/m$$

With 38 mm x 235 mm SPF joists and 12.5 mm plywood with 3.66 mm nails at 100 mm o.c. along diaphragm boundaries (panel layout case 1), the factored shear strength is

$$v_r = \phi v_d K_D K_{SF} J_{sp} = 0.7 \times 10.5 \times 1.15 \times 1.0 \times 0.8 = 6.76\ kN/m > 1.48\ kN/m$$

Design example: Wood diaphragm using envelope method